

# $\alpha'$ -expansion of Anti-Symmetric Wilson Loops in $\mathcal{N} = 4$ SYM from Fermi Gas

Masaatsu Horikoshi and Kazumi Okuyama

Department of Physics, Shinshu University,  
Matsumoto 390-8621, Japan

## Abstract

We study the large 't Hooft coupling expansion of 1/2 BPS Wilson loops in the anti-symmetric representation in  $\mathcal{N} = 4$  super Yang-Mills (SYM) theory at the leading order in the  $1/N$  expansion. Via AdS/CFT correspondence, this expansion corresponds to the  $\alpha'$  expansion in bulk type IIB string theory. We show that this expansion can be systematically computed by using the low temperature expansion of Fermi distribution function, known as the Sommerfeld expansion in statistical mechanics. We check numerically that our expansion agrees with the exact result of anti-symmetric Wilson loops recently found by Fiol and Torrents.

# 1 Introduction

1/2 BPS circular Wilson loops in 4d  $\mathcal{N} = 4$  super Yang-Mills (SYM) theory are interesting observables which can be computed exactly by a Gaussian matrix model [1, 2, 3]. Via AdS/CFT correspondence, 1/2 BPS Wilson loops in the fundamental representation correspond to the fundamental string in type IIB string theory on  $AdS_5 \times S^5$  [4, 5]. When the rank of the representations becomes large, the corresponding dual objects in the bulk are not fundamental strings but D-branes, and such Wilson loops are sometimes called “Giant Wilson loops”. In particular, 1/2 BPS Wilson loops in the rank  $k$  symmetric and anti-symmetric representations correspond to D3-branes and D5-branes, respectively, with  $k$  unit of electric flux on their worldvolumes [6, 7, 8, 9]. The leading term in the ’t Hooft expansion of  $\mathcal{N} = 4$  SYM side is successfully matched with the DBI action of D-branes in the bulk side. For more general representations, a dictionary between Wilson loops in higher rank representations and bulk D-brane picture was proposed in [10, 11]<sup>1</sup>.

We are interested in the subleading corrections in this correspondence. Recently, there are some progress in the computation of one-loop correction in the  $1/N$  expansion of Giant Wilson loops [14, 15, 16, 17]. Here we will focus on the subleading corrections in the large  $\lambda$  expansion (or  $1/\lambda$  expansion) with  $\lambda$  being the ’t Hooft coupling  $\lambda = g_{YM}^2 N$ , and we will restrict ourselves to the leading order in the  $1/N$  expansion. From the holographic dictionary  $R_{AdS}^2/\alpha' = \sqrt{\lambda}$ , the large  $\lambda$  expansion on the SYM side corresponds to the  $\alpha'$  expansion in the bulk string theory side.

In this paper, we consider the large  $\lambda$  expansion of the 1/2 BPS Wilson loops in the anti-symmetric representation. Using the fact that the generating function of anti-symmetric representations can be written as a system of fermions, one can systematically compute the subleading corrections in the large  $\lambda$  expansion by a low temperature expansion of the Fermi distribution function, known as the Sommerfeld expansion. Here the role of temperature is played by  $1/\sqrt{\lambda}$ . We have checked numerically that the subleading corrections agree with the exact expression of the anti-symmetric Wilson loops recently found in [18].

The rest of the paper is organized as follows. In section 2, we find a systematic large  $\lambda$  expansion of anti-symmetric Wilson loops using the Sommerfeld expansion of Fermi distribution function. Our main result is (2.10). In section 3, we compare our result (2.10) with the exact expression in [18], and find a nice agreement. We conclude in section 4 and discuss some future directions.

## 2 Large $\lambda$ expansion of Wilson loops in the anti-symmetric representation

We consider the vacuum expectation value (VEV) of 1/2 BPS circular Wilson loops in  $\mathcal{N} = 4$  SYM with gauge group  $U(N)$ . After applying the supersymmetric localization [3], the Wilson loop VEV is reduced to a Gaussian matrix model

$$\left\langle \text{Tr}_R P \exp \left[ \oint ds \left( i A_\mu \dot{x}^\mu + \Phi_I \theta^I |\dot{x}| \right) \right] \right\rangle = \int dM e^{-\frac{1}{2\pi g_s} \text{Tr} M^2} \text{Tr}_R(e^M). \quad (2.1)$$

Here  $x^\mu(s)$  parametrizes a great circle of  $S^4$  on which  $\mathcal{N} = 4$  SYM lives, and  $\Phi_I$  ( $I = 1, \dots, 6$ ) denote the adjoint scalar fields in  $\mathcal{N} = 4$  SYM and  $\theta^I \in S^5$  is a constant unit vector.  $g_s$  in (2.1)

---

<sup>1</sup> Bubbling geometries including the effect of back-reaction due to the insertion of Wilson loops were studied in [12, 13].

denotes the string coupling which is related to the Yang-Mills gauge coupling  $g_{\text{YM}}$  by

$$g_s = \frac{g_{\text{YM}}^2}{4\pi}. \quad (2.2)$$

In this paper, we will focus on the Wilson loop VEV in the  $k^{\text{th}}$  anti-symmetric representation  $R = A_k$

$$W_{A_k} = \int dM e^{-\frac{1}{2\pi g_s} \text{Tr} M^2} \text{Tr}_{A_k}(e^M). \quad (2.3)$$

It is convenient to define the VEV of  $SU(N)$  part by removing the  $U(1)$  contribution

$$\mathcal{W}_{A_k} = W_{A_k} e^{-\frac{\pi k g_s}{2}}. \quad (2.4)$$

One can show that  $\mathcal{W}_{A_k}$  is symmetric under  $k \rightarrow N - k$

$$\mathcal{W}_{A_{N-k}} = \mathcal{W}_{A_k}. \quad (2.5)$$

We are interested in the behavior of Wilson loop VEV  $\mathcal{W}_{A_k}$  in the limit

$$N \rightarrow \infty \quad \text{with} \quad \lambda = g_{\text{YM}}^2 N, \quad \frac{k}{N} \text{ fixed}. \quad (2.6)$$

In the large  $\lambda$  limit together with (2.6), the anti-symmetric Wilson loop  $\mathcal{W}_{A_k}$  is holographically dual to a D5-brane in  $AdS_5 \times S^5$ , whose worldvolume has the form  $AdS_2 \times S^4$  [7]. From the computation of the DBI action of D5-brane, the leading behavior of  $\mathcal{W}_{A_k}$  is found to be

$$\log \mathcal{W}_{A_k} = \frac{2N\sqrt{\lambda}}{3\pi} \sin^3 \theta_k = \frac{1}{g_s} \frac{(\sqrt{\lambda} \sin \theta_k)^3}{6\pi^2}, \quad (2.7)$$

where  $\theta_k$  is given by

$$\theta_k - \sin \theta_k \cos \theta_k = \frac{\pi k}{N}. \quad (2.8)$$

From the bulk D5-brane picture, the angle  $\theta_k$  parametrizes the position of  $S^4$  part of the world-volume inside the  $S^5$  of bulk geometry  $AdS_5 \times S^5$ .

We are interested in the subleading corrections of  $\mathcal{W}_{A_k}$ . There are two expansion parameters  $g_s$  and  $1/\lambda$ . In [14], it was reported that the one-loop correction in the  $g_s$  expansion has the form

$$\log \mathcal{W}_{A_k} = \frac{1}{g_s} \frac{(\sqrt{\lambda} \sin \theta_k)^3}{6\pi^2} + c \log \sin \theta_k, \quad (2.9)$$

where  $c$  is an order 1 constant. In this paper, we will consider subleading corrections of the  $1/\lambda$  expansion while we focus on the leading order in the  $g_s$ -expansion.

As we will show below, the  $1/\lambda$  expansion of  $\mathcal{W}_{A_k}$  can be computed as

$$\begin{aligned} \log \mathcal{W}_{A_k} = \frac{1}{g_s} \left[ \frac{(\sqrt{\lambda} \sin \theta_k)^3}{6\pi^2} + \frac{\sqrt{\lambda} \sin \theta_k}{12} - \frac{\pi^2(19 + 5 \cos 2\theta_k)}{\sqrt{\lambda} \sin^3 \theta_k} \right. \\ \left. - \frac{\pi^4(6788 \cos 2\theta_k + 35 \cos 4\theta_k + 8985)}{362880 \lambda^{\frac{3}{2}} \sin^7 \theta_k} + \dots \right]. \end{aligned} \quad (2.10)$$

This is our main result.

Let us explain how we obtained (2.10). To study the anti-symmetric Wilson loops systematically, it is convenient to introduce the generating function of  $\mathcal{W}_{A_k}$  by summing over  $k$  with fugacity  $e^\mu$

$$\sum_{k=0}^N e^{k\mu} \mathcal{W}_{A_k} = \langle \det(1 + e^\mu e^M) \rangle_{mm} \quad (2.11)$$

where  $\langle \mathcal{O} \rangle_{mm}$  denotes the expectation value in the Gaussian matrix model

$$\langle \mathcal{O} \rangle_{mm} = \int dM e^{-\frac{1}{2\pi g_s} \text{Tr} M^2} \mathcal{O}. \quad (2.12)$$

Using the large  $N$  factorization we find

$$\langle \det(1 + e^\mu e^M) \rangle_{mm} \approx e^{\langle \text{Tr} \log(1 + e^\mu e^M) \rangle_{mm}} \quad (2.13)$$

up to  $1/N$  corrections, and the right hand side of (2.13) in the planar limit becomes

$$\langle \text{Tr} \log(1 + e^\mu e^M) \rangle_{mm} = N \int_{-\sqrt{\lambda}}^{\sqrt{\lambda}} dm \rho(m) \log(1 + e^{\mu-m}), \quad (2.14)$$

where  $\rho(m)$  is the Wigner semi-circle distribution of Gaussian matrix model

$$\rho(m) = \frac{2}{\pi\lambda} \sqrt{\lambda - m^2}. \quad (2.15)$$

Then, as discussed in [8], the Wilson loop VEV in the  $k^{\text{th}}$  anti-symmetric representation is written as an integral over the chemical potential  $\mu$

$$\mathcal{W}_{A_k} = \int d\mu \exp \left[ -k\mu + N \int_{-\sqrt{\lambda}}^{\sqrt{\lambda}} dm \rho(m) \log(1 + e^{\mu-m}) \right]. \quad (2.16)$$

By rescaling  $(m, \mu) \rightarrow (\sqrt{\lambda}m, \sqrt{\lambda}\mu)$ , we can further rewrite (2.16) as

$$\mathcal{W}_{A_k} = \int d\mu \exp \left[ N \left( -\frac{k}{N} \sqrt{\lambda} \mu + J(\mu) \right) \right], \quad (2.17)$$

where

$$J(\mu) = \frac{2}{\pi} \int_{-1}^1 dm \sqrt{1 - m^2} \log(1 + e^{\sqrt{\lambda}(\mu-m)}). \quad (2.18)$$

In the regime of our interest (2.6), the  $\mu$ -integral in (2.17) can be evaluated by the saddle point approximation since the exponent in (2.17) is multiplied by the large number  $N$ . Thus we conclude that  $\log \mathcal{W}_{A_k}$  is essentially given by the Legendre transform of  $J(\mu)$

$$\log \mathcal{W}_{A_k} = -k\sqrt{\lambda}\mu_* + NJ(\mu_*), \quad (2.19)$$

where  $\mu_*$  is determined by the saddle point equation

$$\partial_\mu J(\mu) \Big|_{\mu=\mu_*} = \frac{k\sqrt{\lambda}}{N}. \quad (2.20)$$

Note that the fluctuation of  $\mu$ -integral around the saddle point gives rise to a subleading correction in  $g_s$ , as in the case of ABJM Fermi gas [19], and hence we can safely ignore such corrections for our purpose to study the leading order behavior in the  $g_s$  expansion<sup>2</sup>.

Noticing that the Fermi distribution function naturally appears in the derivative of  $J(\mu)$

$$\partial_\mu J(\mu) = \frac{2\sqrt{\lambda}}{\pi} \int_{-1}^1 dm \frac{\sqrt{1-m^2}}{1+e^{\sqrt{\lambda}(m-\mu)}}, \quad (2.21)$$

one can easily compute the  $1/\lambda$  expansion by the standard Sommerfeld expansion in statistical mechanics, where  $1/\sqrt{\lambda}$  plays the role of temperature.

The large  $\lambda$  expansion of Fermi distribution function reads

$$\begin{aligned} \frac{1}{1+e^{\sqrt{\lambda}(m-\mu)}} &= \frac{\pi \partial_\mu}{\sqrt{\lambda} \sin \frac{\pi \partial_\mu}{\sqrt{\lambda}}} \Theta(\mu - m) \\ &= \sum_{n=0}^{\infty} \frac{(-1)^n B_{2n}(1/2)}{(2n)!} \left( \frac{4\pi^2 \partial_\mu^2}{\lambda} \right)^n \Theta(\mu - m) \\ &= \left( 1 + \frac{\pi^2 \partial_\mu^2}{6\lambda} + \frac{7\pi^4 \partial_\mu^4}{360\lambda^2} + \cdots \right) \Theta(\mu - m) \end{aligned} \quad (2.22)$$

where  $B_{2n}(1/2)$  is the value of Bernoulli polynomial  $B_{2n}(z)$  at  $z = 1/2$ , and  $\Theta(\mu - m)$  is the step-function

$$\Theta(\mu - m) = \begin{cases} 1 & (\mu > m), \\ 0 & (\mu < m). \end{cases} \quad (2.23)$$

Introducing the variable  $\theta$  by

$$\mu = -\cos \theta, \quad (2.24)$$

one can easily show that  $\partial_\mu J(\mu)$  is expanded as

$$\begin{aligned} \partial_\mu J(\mu) &= \frac{2\sqrt{\lambda}}{\pi} \left[ \frac{1}{2}(\theta - \sin \theta \cos \theta) \right. \\ &\quad \left. + \sum_{n=1}^{\infty} \frac{(-1)^n B_{2n}(1/2)}{(2n)!} \left( \frac{4\pi^2}{\lambda} \right)^n \left( \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \right)^{2n-1} \sin \theta \right], \end{aligned} \quad (2.25)$$

from which the expansion of  $J(\mu)$  is found to be

$$\begin{aligned} J(\mu) &= \int d\theta \sin \theta \partial_\mu J(\mu) \\ &= \frac{2\sqrt{\lambda}}{\pi} \left[ \frac{\sin^3 \theta}{3} - \frac{1}{2}(\theta - \sin \theta \cos \theta) \cos \theta \right. \\ &\quad \left. + \sum_{n=1}^{\infty} \frac{(-1)^n B_{2n}(1/2)}{(2n)!} \left( \frac{4\pi^2}{\lambda} \right)^n \left( \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \right)^{2n-2} \sin \theta \right]. \end{aligned} \quad (2.26)$$

Finally, solving the saddle point equation (2.20) order by order in  $1/\lambda$  expansion, and plugging the solution  $\mu_*$  into (2.19), we arrive at our main result (2.10). In this way, we can compute the  $1/\lambda$  expansion of  $\mathcal{W}_{A_k}$  up to any desired order.

---

<sup>2</sup>The overall constant of the integral (2.16) and the factor coming from the change of variable  $\mu \rightarrow \sqrt{\lambda}\mu$  from (2.16) to (2.17) are also subleading in the  $g_s$  expansion, and we simply ignore them as well.

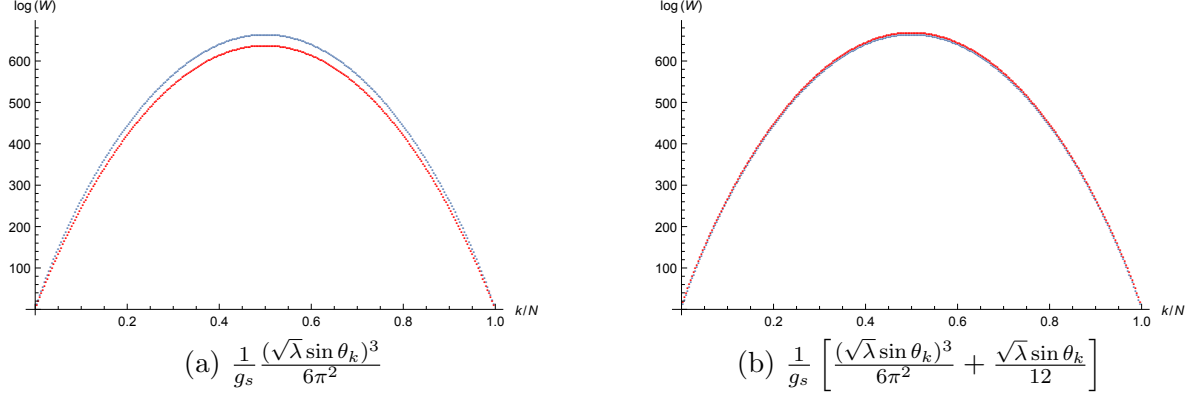


Figure 1: This is the plot of  $\log \mathcal{W}_{A_k}$  as a function of  $k/N$ , for  $N = 300$ ,  $\lambda = 100$ . The blue dots are the exact values obtained from (3.1), while the red dots represent the behavior of (a) leading term only (b) leading + next-to-leading terms in the expansion (2.10). One can clearly see that the inclusion of the next-to-leading correction improves the matching.

### 3 Comparison with the exact result

Let us compare our result (2.10) with the exact result of anti-symmetric Wilson loops at finite  $N$  and  $k$  found in [18].

It is found in [18] that the generating function of  $\mathcal{W}_{A_k}$  (2.11) is exactly written as a characteristic polynomial of  $N \times N$  matrix  $A$

$$\sum_{k=0}^N e^{k\mu} \mathcal{W}_{A_k} = \det(1 + e^\mu A), \quad (3.1)$$

where the matrix element  $A_{i,j}$  is given by the generalized Laguerre polynomial

$$A_{i,j} = L_{i-1}^{j-i}(-\pi g_s), \quad (i, j = 1, \dots, N). \quad (3.2)$$

From this expression (3.1), one can extract the exact value of Wilson loop VEV  $\mathcal{W}_{A_k}$  at arbitrary value of  $N, k$  and  $g_s$ .

In Fig.1, we show the plot of  $\log \mathcal{W}_{A_k}$  as a function of  $k/N$  for  $N = 300$  and  $\lambda = 100$ , corresponding to the value of string coupling  $g_s = \frac{\lambda}{4\pi N} = \frac{1}{12\pi}$ . The blue dots are the exact values obtained from (3.1) while the red dots are the plot of our result (2.10) for the leading term (Fig.1(a)) and the leading + next-to-leading terms (Fig.1(b)). One can clearly see that the leading + next-to-leading terms in Fig.1(b) exhibit a nice agreement with the exact result (3.1). Interestingly, the leading term alone is not enough to reproduce the behavior of the exact result (3.1), and the next-to-leading correction has a rather large contribution for this choice of parameters  $N = 300, \lambda = 100$ . Note that the leading and the next-to-leading terms in (2.10) are of order  $\lambda^{3/2}/g_s$  and  $\lambda^{1/2}/g_s$ , respectively, while the higher order terms have negative powers of  $\lambda$ , hence in the large  $\lambda$  limit higher order corrections in (2.10) are suppressed. Indeed we have checked that the inclusion of higher order corrections does not change the plot significantly, and the exact result (3.1) is well approximated already at the next-to-leading order. We have performed similar numerical checks for various values of  $N$  and  $\lambda$  ( $N, \lambda \gg 1$ ) and find a good agreement for all cases.

## 4 Conclusion

We have computed the  $1/\lambda$  expansion (or  $\alpha'$ -expansion of bulk type IIB string theory) of Wilson loop VEV in the anti-symmetric representation using the Sommerfeld expansion of Fermi distribution function. It would be very interesting to reproduce this result from the computation of  $\alpha'$ -correction of the D5-brane action in the  $AdS_5 \times S^5$  background.

There are many things to be studied further. It is important to develop a method to compute both the  $1/\lambda$  expansion and the  $g_s$  expansion systematically. In particular, it would be interesting to find the  $1/\lambda$  expansion of Wilson loop VEV in the symmetric representation by the low temperature expansion of Bose distribution. However, the integrand of the  $\mu$ -integral might have a singularity corresponding to the onset of Bose-Einstein condensation. It would be interesting to understand the analytic structure of the integrand in the case of symmetric representation (see [8] for a discussion).

Also, it is interesting to understand the convergence property of the expansion. For the  $1/2$  BPS Wilson loop in the fundamental representation, it is observed that the  $\alpha'$ -expansion is *not* Borel summable [2], reflecting the fact that there are corrections of order  $e^{-\sqrt{\lambda}}$  which is non-perturbative in  $\alpha'$ . It would be very interesting to understand the Borel summability of  $\alpha'$ -corrections at fixed  $g_s$  for Wilson loops in various representations. On the other hand, the  $g_s$  expansion of  $1/2$  BPS Wilson loops with fixed  $\lambda$  seems to have a finite radius of convergence<sup>3</sup>, which is consistent with the absence of Yang-Mills instanton corrections to Wilson loop VEV in  $\mathcal{N} = 4$  SYM [3]. We hope that the study of Wilson loops in various representations will provide us with precious information of the non-perturbative structure of string theory.

Finally, we would like to emphasize the importance of our findings. It was reported in [14] that there is a discrepancy between the computation on the field theory side and the string theory side of the one-loop correction in  $1/N$  (2.9) (see [21] for the current status of this problem).<sup>4</sup> However, before settling the issue of this problem of one-loop correction, we have to compute the leading term at large  $N$  as a function of  $\lambda$ , including all  $1/\lambda$  corrections (2.10). After having found the leading term (2.10), one can try to study the one-loop correction in  $1/N$ , either numerically or analytically, by subtracting the leading term from the exact result (3.1). In this computation, it is important to subtract the subleading terms in  $1/\lambda$  since they are rather large and cannot be neglected, as we have seen in section 3. We believe that our result (2.10) is an important first step to resolve the issue of the one-loop discrepancy. We leave the study of one-loop correction as an interesting future problem.

## Acknowledgments

The work of K.O. was supported in part by JSPS KAKENHI Grant Number 16K05316, and JSPS Japan-Hungary and Japan-Russia bilateral joint research projects.

## References

- [1] J. K. Erickson, G. W. Semenoff and K. Zarembo, “Wilson loops in  $N=4$  supersymmetric Yang-Mills theory,” Nucl. Phys. B **582**, 155 (2000) doi:10.1016/S0550-3213(00)00300-X [hep-th/0003055].

---

<sup>3</sup>For the Wilson loop in the fundamental representation, we have checked the convergence of  $g_s$  expansion numerically using the result in [20].

<sup>4</sup>Actually, to understand the origin of this discrepancy was one of the motivation of this work.

- [2] N. Drukker and D. J. Gross, “An Exact prediction of  $N=4$  SUSYM theory for string theory,” *J. Math. Phys.* **42**, 2896 (2001) doi:10.1063/1.1372177 [hep-th/0010274].
- [3] V. Pestun, “Localization of gauge theory on a four-sphere and supersymmetric Wilson loops,” *Commun. Math. Phys.* **313**, 71 (2012) doi:10.1007/s00220-012-1485-0 [arXiv:0712.2824 [hep-th]].
- [4] J. M. Maldacena, “Wilson loops in large  $N$  field theories,” *Phys. Rev. Lett.* **80**, 4859 (1998) doi:10.1103/PhysRevLett.80.4859 [hep-th/9803002].
- [5] S. J. Rey and J. T. Yee, “Macroscopic strings as heavy quarks in large  $N$  gauge theory and anti-de Sitter supergravity,” *Eur. Phys. J. C* **22**, 379 (2001) doi:10.1007/s100520100799 [hep-th/9803001].
- [6] N. Drukker and B. Fiol, “All-genus calculation of Wilson loops using D-branes,” *JHEP* **0502**, 010 (2005) doi:10.1088/1126-6708/2005/02/010 [hep-th/0501109].
- [7] S. Yamaguchi, “Wilson loops of anti-symmetric representation and D5-branes,” *JHEP* **0605**, 037 (2006) doi:10.1088/1126-6708/2006/05/037 [hep-th/0603208].
- [8] S. A. Hartnoll and S. P. Kumar, “Higher rank Wilson loops from a matrix model,” *JHEP* **0608**, 026 (2006) doi:10.1088/1126-6708/2006/08/026 [hep-th/0605027].
- [9] K. Okuyama and G. W. Semenoff, “Wilson loops in  $N=4$  SYM and fermion droplets,” *JHEP* **0606**, 057 (2006) doi:10.1088/1126-6708/2006/06/057 [hep-th/0604209].
- [10] J. Gomis and F. Passerini, “Holographic Wilson Loops,” *JHEP* **0608**, 074 (2006) doi:10.1088/1126-6708/2006/08/074 [hep-th/0604007].
- [11] J. Gomis and F. Passerini, “Wilson Loops as D3-Branes,” *JHEP* **0701**, 097 (2007) doi:10.1088/1126-6708/2007/01/097 [hep-th/0612022].
- [12] S. Yamaguchi, “Bubbling geometries for half BPS Wilson lines,” *Int. J. Mod. Phys. A* **22**, 1353 (2007) doi:10.1142/S0217751X07035070 [hep-th/0601089].
- [13] O. Lunin, “On gravitational description of Wilson lines,” *JHEP* **0606**, 026 (2006) doi:10.1088/1126-6708/2006/06/026 [hep-th/0604133].
- [14] A. Faraggi, J. T. Liu, L. A. Pando Zayas and G. Zhang, “One-loop structure of higher rank Wilson loops in AdS/CFT,” *Phys. Lett. B* **740**, 218 (2015) [arXiv:1409.3187 [hep-th]].
- [15] E. I. Buchbinder and A. A. Tseytlin, “ $1/N$  correction in the D3-brane description of a circular Wilson loop at strong coupling,” *Phys. Rev. D* **89**, no. 12, 126008 (2014) doi:10.1103/PhysRevD.89.126008 [arXiv:1404.4952 [hep-th]].
- [16] A. Faraggi, W. Mueck and L. A. Pando Zayas, “One-loop Effective Action of the Holographic Antisymmetric Wilson Loop,” *Phys. Rev. D* **85**, 106015 (2012) doi:10.1103/PhysRevD.85.106015 [arXiv:1112.5028 [hep-th]].
- [17] A. Faraggi and L. A. Pando Zayas, “The Spectrum of Excitations of Holographic Wilson Loops,” *JHEP* **1105**, 018 (2011) doi:10.1007/JHEP05(2011)018 [arXiv:1101.5145 [hep-th]].
- [18] B. Fiol and G. Torrents, “Exact results for Wilson loops in arbitrary representations,” *JHEP* **1401**, 020 (2014) [arXiv:1311.2058 [hep-th]].



- [19] M. Marino and P. Putrov, “ABJM theory as a Fermi gas,” *J. Stat. Mech.* **1203**, P03001 (2012) doi:10.1088/1742-5468/2012/03/P03001 [arXiv:1110.4066 [hep-th]].
- [20] K. Okuyama, “t Hooft expansion of 1/2 BPS Wilson loop,” *JHEP* **0609**, 007 (2006) doi:10.1088/1126-6708/2006/09/007 [hep-th/0607131].
- [21] K. Zarembo, “Localization and AdS/CFT Correspondence,” arXiv:1608.02963 [hep-th].